CSAT NOTES

1. $20, \overset{\times 0.5}{10}, \overset{\times 1}{10}, \overset{\times 1.5}{15}, \overset{\times 2}{30}, \overset{\times 2.5}{75}, \overset{\times 3}{X}, ?$

$\therefore X = 3 \times 25 = \underline{225}$

2. **Dates**

normal years NY = 365 days $\therefore$ no. of odd days = $7\overline{)365}\ 52$ remainder 1 = 1 day

leap years LY = 366 days → odd days = 2

criteria – i) century years (100th) must be divisible by 400

ii) other yrs. → div. by 4 (00)

Find the total no. of odd days btw 2 desired dates to find which day.

eg: 5th June, 12th June: 12 − 5 = 7 = 0 OD

3. **Simple Interest (SI)** (annual interest rate)

$SI = \frac{P \times R \times T}{100}$ where R = rate in %, P = principal amount, T = time in years.

(final) Amount = P + Interest

4. **Compound Interest (CI)**

Amount, $A = P\left(1 + \frac{r}{n}\right)^{nt}$; r = interest rate (decimal), n = no. of times interest is compounded/yr., t = time (years)

$CI = A - P$

5. **AP (Arithmetic Progression)**: $a, a+d, a+2d, \ldots, a+(n-1)d$

Sum, $S = \frac{n}{2}(2a + (n-1)d) = \frac{n}{2}(a+l)$; d = common difference

6. **GP (Geometric Progression)**: $a, ar, ar^2, \ldots ar^{n-1}$

Sum of GP, $S = \frac{a(r^n - 1)}{r-1}$ or $\frac{a(1-r^n)}{1-r}$; $r \neq 1$; r = common ratio

n = no. of terms, a = 1st term

Sum of infinite GP, $S = \frac{a}{1-r}$, where $|r| < 1$

7. Sum of 'n' natural numbers, $S = \frac{n(n+1)}{2}$

8. Sum of square of 'n' natural numbers, $S = \frac{n(n+1)(2n+1)}{6}$

9. Sum of cube of 'n' natural numbers, $S = \left(\frac{n(n+1)}{2}\right)^2$

10. $AH = G^2$, i.e. A, G, H are in GP; A, G, H are the arithmetic, geometric & harmonic means. $A \geq G \geq H$;

11. **Divisibility Rules**

2 – last digit is 0, 2, 4, 6 or 8.

3 – sum of digits is divisible by 3

4 – last two digits are divisible by 4.

5 - last digit is 0 or 5
6 - no. is divisible by 2 and 3.
7 - using rule of triplet, if alternating sum is divisible by 7.
8 - last 3 digits are divisible by 8.
9 - sum is divisible by 9.
10 - unit digit is 0.
11 - difference of odd digits & even digits are 0 or divisible by 11.
12 - no. is divisible by 3 & 4.
13 - using rule of triplet, if alternating sum is divisible by 13.

12. Solving for 3 unknown terms in AP: $a-d, a, a+d$; 4: $a-3d, a-d, a+d, a+3d$
13. Arithmetic mean $= \frac{x_1+x_2+\dots+x_n}{n}$; n = no. of observations
14. Geometric mean $= \sqrt[n]{x_1 x_2 \dots x_n}$
15. 3 unknowns: $\frac{a}{r}, a, ar$
16. Harmonic mean $= \frac{n}{\frac{1}{x_1}+\frac{1}{x_2}+\dots+\frac{1}{x_n}}$

17. Important algebraic formulas

i) $(a+b)^2 = a^2+b^2+2ab$ ii) $(a-b)^2 = a^2+b^2-2ab$
iii) $a^2-b^2 = (a+b)(a-b)$ iv) $(a+b+c)^2 = a^2+b^2+c^2+2ab+2bc+2ca$
v) $(a+b)^3 = a^3+b^3+3ab(a+b)$ vi) $(a-b)^3 = a^3-b^3-3ab(a-b)$
vii) $a^3+b^3 = (a+b)(a^2-ab+b^2)$ viii) $a^3-b^3 = (a-b)(a^2+ab+b^2)$

18. i) Speed $= \frac{\text{distance}}{\text{time}}$; ii) average speed $= \frac{\text{total distance}}{\text{total time}}$
iii) Relative velocity, $v_{12} = \bar{v}_1 - \bar{v}_2$
iv) kmph → m/s : multiply by $5/18$
v) speed downstream = speed of boat in still water + speed of stream
speed upstream = speed of boat in still water - speed of stream

19. Simple and Compound interest:

i) Doubling of money - Rule of 72
rate × time = 72 (approx)
ii) Tripling of money - Rule of 114: rate × time = 114
iii) Quadrupling of money - Rule of 144: rate × time = 144

20. **Profit and Loss:**

i) profit = SP − CP = selling price − cost price

ii) profit % = $\frac{\text{Profit}}{\text{CP}} \times 100\%$ iii) discount = MP − SP ; MP = market price or marked price

iv) discount % = $\frac{\text{discount}}{\text{MP}} \times 100\%$

21. **Averages:**

Weighted average = $\frac{w_1x_1 + w_2x_2 + \dots + w_nx_n}{x_1 + x_2 + \dots + x_n}$

22. % change = $\frac{\text{Final value} - \text{Initial value}}{\text{Initial value}} \times 100\%$

23 **LCM & HCF:**

i) HCF of fraction = $\frac{\text{HCF of numerator}}{\text{LCM of denominator}}$

ii) LCM of fraction = $\frac{\text{LCM of numerator}}{\text{HCF of denominator}}$

iii) LCM × HCF = product of the two numbers

HCF: 144, 160

144 | 160 (1) — 144 — 16 | 144 (9) — 144 — 0

∴ HCF = 16

LCM: 60, 45

2 | 60, 45
2 | 30, 45
3 | 15, 45
3 | 5, 15
5 | 5, 5
1, 1, 1

∴ LCM = 2×2×3 ×3×5 = 180

iv) The least no. which when divided by a, b and c leaved a remainder R in each case = (LCM of a, b, c) + R

v) The greatest no. which divides a, b and c to leave the remainder R = HCF{a−R, b−R, c−R}

vi) The greatest no. which divides a, b & c to leave remainders x, y & z = HCF{a−x, b−y, c−z}

24. **Factorials** (0! = 1)

i) $n! = 1 \times 2 \times 3 \times \dots \times n$ ii) $n! = n(n-1)!$

iii) Permutation, ${}^nP_r = \frac{n!}{(n-r)!}$ iv) Combination, ${}^nC_r = \frac{n!}{r!(n-r)!}$

v) ${}^nC_r = {}^nC_{n-r}$ vi) ${}^nC_0 + {}^nC_1 + {}^nC_2 + \dots + {}^nC_n = 2^n$

vii) ${}^nC_0 = 1 = {}^nC_n$ viii) ${}^nC_1 = n = {}^nC_{n-1}$

ix) Total no. of handshakes possible among n people = ${}^nC_2$

x) Total no. of triangles that can be formed by joining sides of polygon of n sides = ${}^nC_3$.

xi) Total no. of diagonals of a polygon of n sides = $\frac{n(n-3)}{2}$

xii) Total no. of circular permutations if clockwise and anti-clockwise are taken as different = $(n-1)!$, else → $\frac{(n-1)!}{2}$

0! = 1	4! = 24
1! = 1	5! = 120
2! = 2	6! = 720
3! = 6	7! = 5040

25. **Probability**

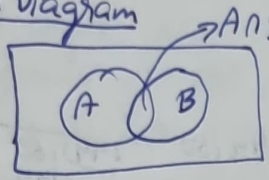
i) Probability, P(not A) = 1 − P(A) ii) P of an event = $\frac{\text{no. of favourable outcomes}}{\text{total no of outcomes}}$

iii) Odds in favour of an event = $\frac{\text{no. of favourable outcomes}}{\text{no. of unfavourable outcomes}}$

iv) Odds against an event = $\frac{\text{no. of unfavourable outcomes}}{\text{no. of favourable outcomes}}$

v) $P(A \cap B) = P(A)P(B)$ if A & B are independent events.

26. Venn Diagram



i) $n(A \cup B) = n(A) + n(B) - n(A \cap B)$

ii) $n(A \cup B \cup C) = n(A) + n(B) + n(C) - n(A \cap B) - n(B \cap C) - n(A \cap C) + n(A \cap B \cap C)$

27. Quadratic Equation:

i) $ax^2 + bx + c = 0 \Rightarrow x = \frac{-b \pm \sqrt{b^2 - 4ac}}{2a}$

ii) $x^2 - Sx + P = 0$; S = sum of roots $= x_1 + x_2$; P = product of roots $= x_1 x_2$

$\therefore S = -\frac{b}{a}$, $P = \frac{c}{a}$

28. Important series sum:

i) Sum of first n odd numbers $= n^2$ $(\in N)$

ii) Sum of first n even numbers $= n(n+1)$

29. Ratios and Proportions

$\frac{a}{b} = \frac{c}{d} \Rightarrow$ i) $\frac{a+b}{b} = \frac{c+d}{d}$ ii) $\frac{a-b}{b} = \frac{c-d}{d}$ iii) $\frac{a+b}{a-b} = \frac{c+d}{c-d}$

iv) $\frac{a-c}{b-c} < \frac{a}{b} < \frac{a+c}{b+c}$ (when $a<b$, else opposite relation) (i.e. proper fraction)

30. Surds and Indices

i) $x^0 = 1$ ii) $x^a x^b = x^{a+b}$ iii) $x^{-a} = \frac{1}{x^a}$ iv) $\sqrt[a]{x} = x^{1/a}$

v) $(x^a)^b = x^{ab}$ vi) $x^a y^a = (xy)^a$

31. Squares

1 - 1	9 - 81	17 - 289	25 - 625
2 - 4	10 - 100	18 - 324	26 - 676
3 - 9	11 - 121	19 - 361	27 - 729
4 - 16	12 - 144	20 - 400	28 - 784
5 - 25	13 - 169	21 - 441	29 - 841
6 - 36	14 - 196	22 - 484	30 - 900
7 - 49	15 - 225	23 - 529	31 - 961
8 - 64	16 - 256	24 - 576	32 - 1024

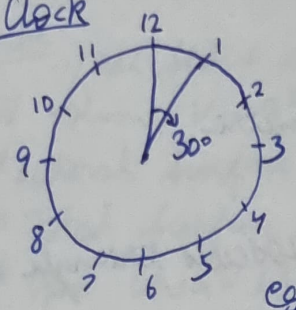
33. Cubes

1 - 1	5 - 125	9 - 729
2 - 8	6 - 216	10 - 1000
3 - 27	7 - 343	11 - 1331
4 - 64	8 - 512	12 - 1728

34. Calendar

i) normal year = 365 days = 52 weeks + 1 day
ii) leap year = 366 days = 52 weeks + 2 days
iii) ~~No~~ no of leap years in 100 year time period = 24 or 25 depending on whether 100 year is leap year or not.
iv) leap years in 400 year time period = 97 = 24×3 + 25

35. Clock



i) Exact minutes when a particular angle will be formed b/w hour hand and minute hand.

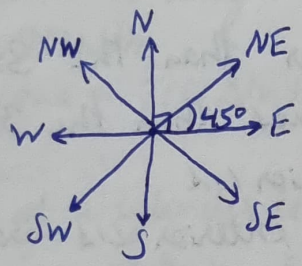
min. = $\frac{2}{11}(hi \pm \theta)$; where hi = smaller hour no. $\times 30$;

θ = required angle

eg: At what time b/w 7 & 8 o'clock, are the hands of a clock at an angle of 45°? ∴ time = 7; min = $\frac{2}{11}(7^{\times 30} \pm 45)$

∴ min = $\frac{2}{11}(210 \pm 45) = 30, 46\frac{4}{11}$; ∴ Time = 7:30 & 7:46 $\frac{4}{11}$

36. Direction



36. Numbers

i) Representation of a no. $ab = \cancel{100}\ 10a + b$; $abc = 100a + 10b + c$
ii) Whole no.'s = Natural no.'s + {0}
iii) Rational no. = can be expressed in terms of p/q
iv) Irrational no. = cannot be expressed in terms of p/q.
v) proper fraction = no.'s in the form of p/q where p < q.
vi) Improper fraction = no.'s in the form of p/q where p > q.
vii) Co-prime no.'s = no.'s ~~not~~ with HCF = 1
viii) composite no. = no. having > 2 factors
ix) Twin prime = pair of prime no.'s where difference = 2; eg: (3,5), (5,7) etc.

32. Unit digit when a no. is raised to some power (EN)

0 - always 0
1 - always 1
2 - 2, 4, 8, ~~8~~ 6
3 - ~~4~~ 3, 9, ~~7~~, 1
4 - 4, 6
5 - always 5
6 - always 6
7 - ~~3, 9~~ 7, 9, 3, 1
8 - ~~2, 4, 6, 8~~ 8, 4, 2, 6 ∴ cyclicity = 4
9 - ~~9, 9~~ 9, 1 - cyclicity = 2

eg: Find unit digit in $(4167)^{434}$.
↳ Then simply check for 7. cyclicity = 4 {7, 9, 3, 1} $434 \div 4 = 2$ ∴ U.D. = 9

38. Mean, Median and Mode

i) mean = $\frac{\text{sum of observations}}{\text{no. of observations}}$

ii) median = middlemost observation when data is arranged in ascending or descending order. (even no's → $\frac{n_m + n_{m+1}}{2}$)

iii) mode = most frequently occuring value in data set

iv) 2mean + mode = 3median

v) Range = max. value − min. value

vi) Skewed data = data sets that are not symmetric

39. Triangles

i) Equilateral Δ ⇒ △ ii) ~~Isosceles~~ ~~Isosceles~~ Isosceles triangle △

iii) Scalene Δ → no. equal sides

iv) Acute Δ → all ∠'s < 90° v) Right Δ → one ∠ = 90°; $p^2 + b^2 = h^2$

vi) Obtuse Δ → one ∠ > 90°

vii) Sum of 3 ∠s of a Δ = 180°

viii) Sum of 2 sides is always greater than the 3rd side.

ix) Difference of 2 sides is always less than the 3rd side.

x) Exterior ∠ = sum of 2 opposite interior ∠s

xi) Sum of interior ∠'s = 180° xii) Sum of exterior ∠'s = 360°

xiii) Perimeter of Δ = sum of (length of) sides = a + b + c

xiv) semi-perimeter, s = $\frac{\text{perimeter}}{2}$

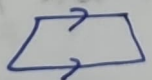
xv) Area of a Δ using Heron's formula, $A = \sqrt{s(s-a)(s-b)(s-c)}$

xvi) Area of a Δ = $\frac{1}{2}$ × base × height = $\frac{1}{2}ab\sin\theta$ where θ = ∠ btw a & b.

40. Quadrilateral

i) total no. of sides = 4 ii) sum of interior ∠'s = 360°

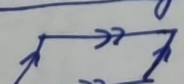
a) Trapezium



i) 2 sides are parallel to each other, & 2 are non-parallel

ii) Area, A = $\frac{1}{2}$(sum of parallel sides) × height

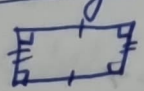
b) Parallelogram i) opposite sides are equal and parallel



ii) opposite angles are equal.

iii) perimeter = 2(a+b) iv) Area, A = $\frac{1}{2}$ × product of diagonals × sin θ (θ = ∠ btw sides of ||gram) also, ii) A = base × height iii) A = ab sin θ (θ = ∠ btw diagonals)

c) Rectangle

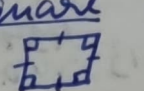


i) opposite sides are equals parallel; all ∠'s are 90°

ii) perimeter, P = 2(l + b) iii) Area, A = l × b

iii) diagonal = $\sqrt{l^2 + b^2} = D_1 = D_2$

d) Square

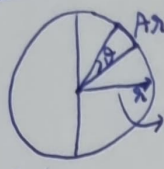


i) All sides are equal; ii) All ∠'s are 90°

iii) P = 4a iv) A = a^2 v) D = $\sqrt{2}a$

c) Rhombus: i) All sides are equal ii) Opposite ∠'s are equal
iii) P = 4a iv) A = ½ × product of diagonals = base × height
v) diagonals intersect at 90° vi) Also, A = base × height = $a^2 \sin\theta$

41. Circle



i) diameter, d = 2r ; ii) Circumference = $2\pi r$ → θ in degrees
iii) $A = \pi r^2$ iv) Area of arc = $\frac{\theta}{360} \times \pi r^2$
v) length of arc, $l = r\theta$ (θ in rad)

42. Cube i) all sides are equal = a
ii) volume, $V = a^3$ iii) length of diagonal = $\sqrt{3}a$
iv) Lateral surface area = perimeter of base × height = $4a^2$
v) Total surface area = $4a^2 + 2 \times$ base area = $6a^2$
vi) Open area = LSA + base area = $5a^2$

43. Cuboid
i) base of cuboid is a rectangle
ii) V = l × b × h iii) length of diagonal = $\sqrt{l^2+b^2+h^2}$
iv) LSA = 2(l+b)h
v) Open area = 2(l+b)h + lb vi) TSA = 2(l+b)h + 2lb = 2(lb + lh + bh)

44. Cylinder
i) base of a cylinder is a circle with radius r, and height h.
ii) Volume = $\pi r^2 \times h$ iii) Curved surface area = $2\pi rh$
iv) TSA = CSA + 2 base area = $2\pi rh + 2\pi r^2$

45. Sphere
i) diameter, d = 2r ii) SA (surface area) = $4\pi r^2$
iii) Volume, $V = \frac{4}{3}\pi r^3$

46. Time and Work
i) days required to complete a work = $\frac{1}{\text{work done in 1 day}}$
ii) Efficiency $\propto \frac{1}{\text{Time taken}}$
iii) If n_1 persons can do w_1 work in d_1 days working T_1 hrs/day with E_1 efficiency and n_2 persons can do w_2 work in d_2 days working T_2 hours each day with E_2 efficiency, then

$$\frac{n_1 d_1 t_1 E_1}{w_1} = \frac{n_2 d_2 t_2 E_2}{w_2}$$

eg: X can complete 20% of work in 8 days. Y can complete 25% of the work in 6 days. If they work together, in how many days will 40% of the work be completed?

Soln X → time to complete 100% work = 40 days.

Y → time to complete 100% work = 24 days

Both together: X+Y → work done in 1 day = $\frac{1}{40} + \frac{1}{24} = \frac{3+5}{120} = \frac{1}{15}$

∴ Time to complete 100% work = 15 days.

⇒ Time to complete 40% work = $0.4 \times 15 =$ <u>6 days</u>

47. <u>Trigonometry</u>

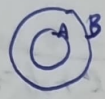
i) $\sin\theta = \frac{p}{h}$ ii) $\cos\theta = \frac{b}{h}$ iii) $\tan\theta = \frac{p}{b}$

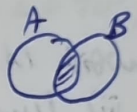
(b = base, p = perpendicular, h = hypotenuse)

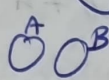
iv)

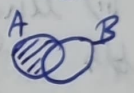
	0°	30°	45°	60°	90°
$\sin\theta$	0	$1/2$	$1/\sqrt{2}$	$\sqrt{3}/2$	1
$\cos\theta$	1	$\sqrt{3}/2$	$1/\sqrt{2}$	$1/2$	0
$\tan\theta$	0	$1/\sqrt{3}$	1	$\sqrt{3}$	not defined ($\to\infty$)

48. <u>Syllogism diagrams</u>

i) All A are B. 

ii) Some A are B 

iii) No A are B 

iv) Some A are not B 

49. <u>Coding - Decoding</u>

A	B	C	D	E	F	G	H	I	J	K	L	M	N	O	P	Q	R	S	T	U	V	W	X	Y	Z
1	2	3	4	5	6	7	8	9	10	11	12	13	14	15	16	17	18	19	20	21	22	23	24	25	26

<u>EJOTY</u> : E=5, J=10, O=15, T=20, Y=25